Introduction

I believe that structuring my lessons around Vygotsky's theory of the Zone of Proximal Development (ZPD) and Hands-on teaching principles will provide students with the best opportunities for cognitive develop within the mathematics classroom. Both of these theories could be placed under Constructivism theories.

The main theory of Vygotsky that I will be using in the classroom is the theory of the Zone of Proximal Development (ZPD). This involves having a higher knowledgeable person teaching ideas/skills to a person that would normally find the taught concept too difficult to comprehend (Chaiklin, 2003). This teaching is to be undertaken with guidance and encouragement from the higher knowledgeable person that will eventually allow the student to develop these concepts on their own. This approach should allow students from disadvantaged backgrounds (i.e. English as a second language / Aboriginal) to be taught not only by the teacher but by students who have a higher understanding of the mathematical principles.

Hands-on learning, in short, means "learning by experience" (Rutherford, 1993, p5). This approach allows students to collect and analysis data before drawing conclusions (Bruder, 1993). Hands-on learning is predominately used when teaching science. However, the principles can be applied to mathematics (although it can be argued that mathematics is a science), with hands-on group learning increasing student involvement, participation in group activities and test scores (Garrity, 1998). Furthermore, hands-on learning should relate the mathematical principles back to 'real world' scenarios to further engage student learning. The approach of relating mathematics back to 'real world' scenarios has been questioned (Kaminski *et al*., 2008). However, previous works have shown that 'real world' learning is an effective teaching strategy for mathematics (Koedinger *et al*., 1997; Podolefsky and Finkelstein, 2007).

Activity one and two (Appendix 1 and 2) are included in the portfolio as it will place students that are at different levels of cognitive development within a social/group setting. This will allow students who are higher in their understanding of the content to guide students who are lower in their understanding of the content (i.e. Vygotsky's ZPD). Studies have shown that forming groups that are same gender will improve cognitive development (Busch, 1995). If possible, groups within these activities will be same sex. To ensure higher end students are not disengaged both activities are designed to challenge the students understanding with questions that will build on their knowledge from previous questions within the activity. This will allow for questions to gradually get harder and not disengage students from the start with the most difficult questions. The combination of activities and small group discussions has also been shown to help with student misconceptions related to the activity being taught (Shaughnessy, 1977) and provides cognitive learning through 'non-traditional' teaching within the mathematics classroom (Yackel *et al*, 1991).

Furthermore, relating the activities back to the 'real world' environment (i.e. mars bar, mode of transport) is allowing students to see how the mathematical theories can be related/implemented in everyday life. As mentioned previously, this has been effective in teaching mathematical principles in the past (Koedinger *et al*., 1997; Podolefsky and Finkelstein, 2007).

Both activity sheets (that are provided with the activity) briefly remind students about some of the prerequisite knowledge that is required to undertake the activities.

The methods listed above have been cited as helping students to overcome problems when teaching probability mathematics to pre-college (i.e. high school and primary school) students (Garfield and Ahlgren, 1988).

Commentary

Activity 1 – Year 8 - 'Chance' / 'Statistics and Probability' (Complementary Events and Probability Lines) : (Appendix 1)

Learning Outcomes

There are 5 main outcomes of activity 1. After this activity, students should be able to:

1) Understand the idea/definition of complementary events.
2) Successfully implement and understand the complementary event equation ($P(\bar{E}) = 1 - P(E)$) in order to find the complementary event of several probability events.
3) Understand that probability can range between zero to one.
4) Work in groups to come to a consensus on the outcome of complementary event questions.
5) Critically analysis and critique peers work.

The activity will provide Year 8 students with the opportunity to learn about 'Chance' within the 'Statistics and Probability' strand of the Australian Mathematics Curriculum. Specifically, the second activity will focus on the Australian Mathematics Curriculum content I.D. number ACMSP204. This will include (as described by the Australian Curriculum):

- identifying the complement of familiar events
- understanding that probabilities range between 0 to 1 and that calculating the probability of an event allows the probability of its complement to be found

Prerequisite knowledge

To engage in the activity students should be able to:

- understand how decimals, fractions and percentages represent the same value

- convert between decimals, fractions and percentages
- know and understand the equation to find the probability of an event
- basic understanding of complementary events (i.e. they are the 'opposite' of the probability of an event)
- comprehend the definition of probability
- basic understanding and comprehension of the English language
- understand the basic mathematical language related to probability

Student misconceptions

One common student misconception is related to the basics of probability. Firstly, students overestimate the likelihood of an event occurring 'by taking into account how well it represents some aspect of the parent population' (Fischbein and Schnarch, 1997, p100). A good example of this misconception is asking which person is most likely to win lotto if one person has the numbers 1-6 while another has 6 random numbers (Fischbein and Schnarch 1997). The main misconception students were found to have was that the person with the random numbers has the greatest chance of winning because their numbers were larger and more variable (Fishbein and Gazit, 1984; Fischbein and Schnarch, 1997).

The misconception of 'representativeness' may also be misconceived by students. For example, students are more likely to think that a coin will land 'HTTHTH' when compared with 'HHHHHH' as 'HTTHTH' follows the distribution of 50/50 (Hirsch and O'Donnell, 2001). Students may also perceive there to be a higher likelihood of an event occurring if the numbers are 'lucky' (Fishbein and Gazit, 1984).

During classroom visits I have also noticed that students fail to understand what the '1' means in the complementary event equation. Additionally, students failed to understand what variables to put within the complimentary event equation, often leaving out one or more event values.

Assessing student learning

Student learning will be assessed through open class discussion at the end of lesson. Furthermore, students will be assessed on their progress via group-teacher interaction throughout the class. Students' progress will also be determined based on correct response/s to teacher or student questions and their ability to justify their response/s to the questions. Furthermore, students' ability to detect inaccuracies in their peer's answers in the open discussion section of the activity will be used to determine student progress. Finally, student worksheets and/or workbooks will be collected and analysed to determine student knowledge and understanding of the topic being taught.

Activity 2 – Year 8 - 'Chance' / 'Statistics and Probability' (Venn Diagrams and Probability Lines) : Appendix 2

Learning Outcomes

There are 7 main outcomes of activity 2. After this activity, students should be able to:

1) Understand how Venn diagrams work in relation to probability.
2) Successfully collect their own data for Venn diagram analysis.
3) Understand that data within a Venn diagram can be used to calculate the probability of an event.
4) Understand that probability ranges between zero to one.
5) Understand the terms 'and', 'or and 'not' in probability/math language.
6) Work in groups to come to a consensus on the outcome of probability questions based on Venn Diagrams.
7) Critically analysis and critique peers work.

Specifically, the first activity will focus on the Australian Mathematics Curriculum content I.D. numbers ACMSP204 and ACMSP292. This will include (as described by the Australian Curriculum):

- using Venn diagrams to calculate probabilities for events, satisfying 'and', 'or' and 'not' conditions
- understanding that representing data in Venn diagrams facilitates the calculation of probabilities
- collecting data to answer the questions using Venn diagrams
- understanding that probabilities range between 0 to 1

Prerequisite knowledge

To engage in the activity students should be able to:

- understand how decimals, fractions and percentages represent the same value
- convert between decimals, fractions and percentages
- know and understand the equation to find the probability of an event
- know and understand the definition of probability
- basic understanding and comprehension of the English language
- Basic understanding of what Venn Diagrams are used for (i.e. for category placement and to compare similarities between categories)
- understand the basic mathematical language related to probability

Student misconceptions

The same misconceptions towards probability and chance as outlined by Fishbein and Gazit (1984), Fischbein and Schnarch (1997) and Hirsch and O'Donnell (2001) in Activity 1 apply to this activity.

Assessing student learning

Student learning will be assessed through open class discussion at the end of lesson. Furthermore, students will be assessed on their progress via group-teacher interaction throughout the class. Students' progress will also be determined based on correct response/s to teacher or student questions and their ability to justify their response/s to the questions. Furthermore, students' ability to detect inaccuracies in their peer's answers in the open discussion section of the activity will be used to determine student progress. Finally, student worksheets and/or workbooks will be collected and analysed to determine student knowledge and understanding of the topic being taught.

Conclusion

Implications for classroom practice

Group work, whole-class learning (via group presentations at the end of each activity) and peer-questioning/assessment are the methods that should be implemented for successful learning outcomes of these activities. This will allow for discussion of the topics with their peers, build on ideas about the topic from others and allow for students to come to a justified consensus on the answers. The activities ability to get students to present their ideas to their class/peers and the ability of students to question their peers work will further allow for critical analysis of the topic by students having to justify their responses and students identifying other student's misconceptions about the mathematical topics.

As mentioned within the introduction, I believe that the theories that will allow for the highest possible learning outcomes for the activities are Vygotsky's theory of the Zone of Proximal Development and 'Hands-on' principles. These theories/principles can be placed under the social constructivist philosophy.

The assessment for these activities should be primarily summative. That is, learning should be assessed based off the students' ability to present and justify their answers to the class and/or group and the ability of students to notice inaccuracies in their peers' answers, with justification as to why the answers are inaccurate. However, formative assessment should also be applied when assessing the worksheets/work books. Certain assessment strategies may work well for some students and not so well for other students. Therefore, this range of assessment will allow for the teacher to determine students' progress without biasing towards certain assessment strategies.

The learning activities may need to be extended across multiple lessons. This will be dependent on the amount of time students require to understand the concepts related to activities.

Learning from the portfolio

The above portfolio provides evidence of my learning in the area of creating activities around the requirements of the Australian Curriculum for teaching mathematics to students from disadvantaged and non-disadvantaged backgrounds. This was shown through the activities ability to start with the basic principles of the mathematical concepts and gradually build on these principles to encourage critical thinking by the students. Placing students from different backgrounds, but the same gender, into groups will also allow students from disadvantaged backgrounds (who may not fully understand the concept) the ability to gain knowledge via other students.

The way of assessing student learning takes on multiple assessment items, giving the teacher the ability to assess students through multiple means. This is vital, as students may respond differently to different assessment items and the ability to have multiple ways of assessing learning will provide the teacher with a greater ability to determine where the student is at with the mathematical concept.

References

Bruder, I. (1993). Redefining science: Technology and the new literacy. *Electronic Learning*, *12*(6), 20-24.

Busch, T. (1996). Gender, group composition, cooperation, and self-efficacy in computer studies. *Journal of Educational Computing Research*, *15*, 125–135.

Chaiklin, S. (2003). The zone of proximal development in Vygotsky's analysis of learning and instruction. *In* Vygotsky's educational theory and practice in cultural context. Cambridge, Cambridge University Press.

Fischbein, E. & Gazit, A. (1984). Does the teaching of probability improve probabilistic intuitions?: An exploratory research study. *Educational Studies in Mathematics*, *15*(1), 1-24.

Fischbein, E. & Schnarch, D. (1997). The evolution with age of probabilistic, intuitively based misconceptions. *Journal for Research in Mathematics Education*, *28*(1), 96-105.

Garrity, C. (1998). *Does the use of hands-on learning, with manipulative, improve the test scores of secondary education geometry students?* Masters Dissertation Thesis, St. Xavier University.

Garfield, J. & Ahlgren, A. (1988). Difficulties in learning basic concepts in probability and statistics: implications for research. *Journal for research in Mathematics Education*, *19*(1), 44-63.

Hirsch, L.S. & O'Donnell, A.M. (2001). Representativeness in statistical reasoning: Identifying and assessing misconceptions. *Journal of Statistics Education* 9(2). Retrieved from http://www.amstat.org/publications/jse/v9n2/hirsch.html.

Kaminski, J.A., Sloutsky, V.M. & Heckler, A.F. (2008). The advantage of abstract examples in learning math. *Science*, *320*, 454-455.

Koedinger, K.R., Anderson, J.R., Hadley, W.H. & Mark, M.A. (1997). Intelligent tutoring goes to school in the big city. *International Journal of Artificial Intelligence in Education*, *8*, 30-43.

Podolefsky, N.S. & Finkelstein, N.D. (2007). Analogical scaffolding and the learning of abstract ideas in physics: An example from electromagnetic waves. *Physical Review Special Topics: Physics Education Research, 3*(1), e010109.

Rutherford, F. J. (1993). Hands-on: A means to an end. *2061 Today, 3*(1), 5.

Shaughnessy, J.M. (1977). Misconceptions of probability: An experiment with a small-group activity-based, model building approach to introductory probability at the college level. *Education Studies in Mathematics, 8*, 285-316.

Yackel, E., Cobb, P. & Wood, T. (1991). Small-group interactions as a source of learning opportunities in second-grade mathematics. *Journal for Research in Mathematics Education*, *22*(5), 390-408.

Appendix 1: Activity 1 Description and Activity 1 Worksheet

Activity 1 – Complementary Events Teachers Instructions

Instructions for Activity

The teacher is to arrange students into groups (four students per group). Where possible, place students from different disadvantaged backgrounds with students from non-disadvantaged backgrounds. Attempt to have same gender groups where possible.

Have students copy down the following table into their book if photocopied tables are not to be provided to the students:

Round Number	Teams	Winner
1	Green vs Blue	
1	Red vs White	
1	Yellow vs Black	
2	Blue vs Red	
2	Green vs Yellow	
2	White vs Black	
3	Blue vs Black	
3	White vs Green	
3	Yellow vs White	

Explain the purpose of the activity to the whole class. Then give each group 6 different coloured balls (green, blue, red, white, yellow, black) and a five cent coin. Explain that each ball represents a sports team (students can name the balls as they please; i.e. green ball could be Raiders, blue ball could be Bulldogs).

The teams are to 'face-off' against each other to determine who will win. Explain that the coin is used to determine who will win the game. Heads will be considered a win while tails will be considered a loss.

Get groups to 'play' the games for their sports team by flipping the coin to determine which teams win and which ones lose. At the end of the activity get students to answer the questions (in their groups) on their activity sheets. Remind students that the equations to complete the activities are on the sheets and that students should read the sheets to each other in their groups before starting the activity sheet.

For the probabilities range between 0 to 1 section of the activity sheet students are to be given graph paper/gridded paper if they are needed to complete the task. The teacher should show how to convert to decimal places and how that is represented on graph paper when students are up to that section.

Students are to be given time to complete the activity sheets. Then, in their groups, they are to present their answers to the class with justification as to why they did what they did when obtaining their answers. Students from the audience are to be asked if they think the answers are correct or incorrect (with justification) (NOTE: allow students from the audience to answer for themselves – do not pick a student at random – if no student picks up on an incorrect answer then ask one of the groups from the audience to explain why the answer is correct or incorrect).

Activity 1 - Activity Sheet

Using the data you have collected we will now find the probability of teams either winning or losing their next game.

Let's do a quick example using a different set of data. A group of 30 students were asked how they travelled to school (i.e. car, bus etc.). It was found that 15 students travelled by car, 10 travelled by bus and 5 travelled by bike. The question was then asked – 'What is the probability that a student will travel to school by car tomorrow?'.

Remember the probability equation:

Probability of event = the number in the event / total number of trials

Therefore, the probability that a student will travel to school by car tomorrow is:

P (student travelling by car) = the number of students travelling by car / total number of students

P (student travelling by car) = 15/30

P (student travelling by car) = 1/2

Can you determine what would be the probability that a student would travel by bus?

Now that you have mastered finding probabilities using let's find out some probabilities from our 'real world' scenarios.

Q1) Find the probabilities from the 'sport' activity you did in class. Don't forget to write your final answer in a sentence.

a) P (Green team wins next game)
b) P (Blue team wins next game)
c) P (Blue and Black team losing the next game)
d) P (Green or White team winning the next game)

Now let us find the complement of those events. The definition of a 'complementary event' is 'the probability that the event will not occur'. This can be done by using the following equation:

$P(\bar{E}) = 1 - P(E)$

In this equation P(E) is the probability of the event. We take 1 from the probability of an event because the probability that an event will occur (P(E)) and the probability of getting the complementary event ($P(\bar{E})$) need to equal 1 (as the chance of getting one or the other will be 100% - or 1 if we use decimal places or fractions).

If we look at our example for the ways students get to school we saw that the probability of getting students travelling to school by car was ½. The complementary event of this would be:

$P(\bar{E}) = 1 - P(E)$

P(Ē) = 1 − $\frac{1}{2}$

P(Ē)= $\frac{1}{2}$

Be sure to write your answer in a sentence.

"The probability of a student not travelling by car to school is ½" or "The complementary event of a student travelling by car to school is $\frac{1}{2}$"

Now let's answer the following questions.

Q2) Find the complementary event from the 'sport' activity you did in class. Don't forget to write your final answer in a sentence.

a) Find the complementary event of the green team winning the next game.
b) Find the complementary event of the blue team losing the next game.
c) Find the complementary event of the blue and black team losing the next game.
d) Find the complementary event of the black team losing the next game.

Excellent. You have found the probability and complimentary events of the above events. All events have a probability range between 0 -1. We can make predictions on the likelihood of an event by looking at where our event falls in the probability range. Below is the probability line, with likelihood of an event below the event value.

0	0.25	0.5	0.75	1
Impossible	Unlikely	Equally likely and unlikely	Likely	Certain

So let us find where our complementary events fall within this probability range. For our example at the start of this worksheet we saw the complementary event value of having a student that travels to school by car was 1/2.

P(Ē) = 1/2

Another way of representing this would be:

P(Ē) = 0.5

This is because 1 divided by 2 equals 0.5. On the probability line 0.5 falls at the 'equally likely and unlikely section' of the line. We would therefore say that 'The complimentary event of having a student that travels to school by car is equally likely and unlikely'.

Now it's your turn. Find where your probability and complimentary values fall on the probability line for questions 1 and 2. Don't forget to write your final answer in a sentence – just like I have done in the example above.

<u>Appendix 2: Activity 2 Description and Activity 2 Worksheet</u>

<u>Activity 2 – Venn diagram Teachers Instructions</u>

Instructions for Activity

The students are to categorise themselves into Venn Diagrams. A large area is needed for this activity. Therefore, students are to go outside to an open area or furniture within the classroom is to be moved (ideally go outside to prevent any WPH&S situations occurring due to movement of durniture). Two rope circles (placed next to each other) are to be laid out in the open area. Circle one is called the 'Mars Bar' circle while circle two is called the 'Crunchie' circle.

This activity requires paper squares with the labels 'Mars Bar' or 'Crunchie' (approximately 60 'Mars Bars' and 60 'Crunchie' squares for a group of 30 students). Place the paper squares into a box or hat. Have each student pick two squares from the box or hat. Students who have picked two 'Mars Bar' squares are to stand in the 'Mars Bar' circle. Students who have picked two 'Crunchie' squares are to stand in the 'Crunchie' circle (Figure 1). There will be students who have picked one 'Mars Bar' and one 'Crunchie' squares. Ask the students where can these students stand and let the students discuss the answer. Once the correct answer has been reached move the two circles so that they sightly overlap (Figure 2). Have the students who picked one 'Mars Bar' and one 'Crunchie' square stand in the overlapping section of the circles. Have the students draw the circle set-up and count how many students are standing in each circle section. Remind the students to label each circle as 'Mars Bar' or 'Crunchie'. Do not move the rope circles / Venn Diagram as it is required for the next part of the activity.

Have the students move out of the circles. Rename the two circles bike and the other car. Have students that travel to school by bike stand in the 'Bike' circle and students that travel by car stand in the 'Car' circle. Have students that travel by both means stand in the section of the circles that overlap. Ask the students "Does anyone else travel to school in any other ways?' Hopefully, students will also travel to school by bus. Ask the students 'Where are the students that travel by bus going to stand?" Get the students to discuss where the other students will stand. Guide the discussion as needed. Once an answer has been reached place another rope circle down as shown in Figure 3. This circle will be labelled 'Bus'. Ask the students to stand where they think they should go. Get students to assist/discuss where the remaining students should stand. Guide the student discussion as needed to get the correct answer. Once all students are standing in the correct place count how many students are in the different sections of the circle (now a complete Venn Diagram). If there are students who take different methods of transport to school then get them to stand outside of the circles. All students travel to school by car, bus or bike then YOU (yes, the teacher) should stand outside of the circle and say that you get to school by motorcycle. Get students to draw the shape, label the circles/Venn Diagram and write how many students go into the different sections of the circles/Venn Diagram. Make sure students write how many people (including you) are outside the circle. The number should be placed just next to the Venn Diagram.

Pack up equipment and have students form groups of 3-4. Where possible, have students from different disadvantaged backgrounds with students from non-disadvantaged backgrounds. Attempt to have same gender groups where possible. Arrange desks so groups can discuss results with each other. Hand each student an activity sheet. Students are to work in their groups to complete the

Venn Diagram activity sheet based on their results from the Venn Diagram activities. Remind students that the equations to complete the activities are on the sheets and that students should read the sheets to each other in their groups before starting the activity sheet. For the probabilities range between 0 to 1 section of the activity sheet students are to be given graph paper/gridded paper if they are needed to complete the task. The teacher should show how to convert to decimal places and how that is represented on graph paper when students are up to that section. Therefore, give each group MAB blocks and grid sheets. Coloured pencils are to be provided get group (three colours) to represent different items (i.e. Mars Bar, Crunchie). Promote students to aid other students in their learning where needed.

Students are to be given time to complete the activity sheets. Then, in their groups, they are to present their answers to the class with justification as to why they did what they did when obtaining their answers. Students from the audience are to be asked if they think the answers are correct or incorrect (with justification) (NOTE: allow students from the audience to answer for themselves – do not pick a student at random – if no student picks up on an incorrect answer then ask one of the groups from the audience to explain why the answer is correct or incorrect).

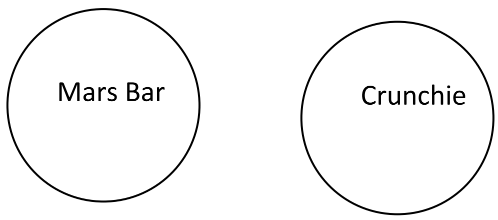


Figure 1

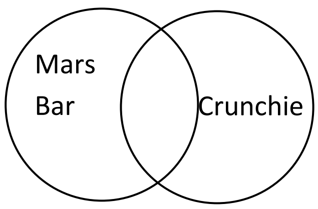


Figure 2

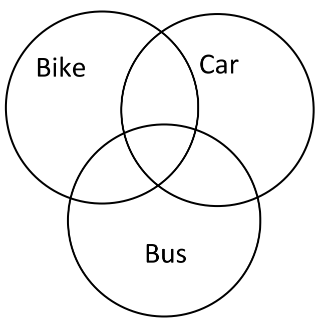
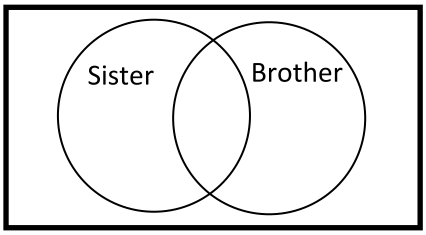


Figure 3

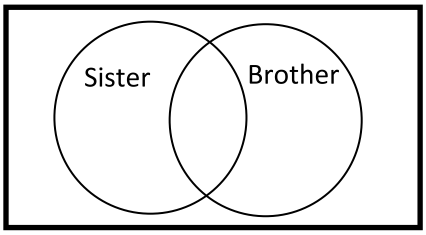
Activity 2 - Activity Sheet

Congratulations. You have just completed Venn-Diagrams using data from 'real world' scenarios. In short, Venn-Diagrams are *diagrams that* use circles to represent sets, with the position and overlap of the circles indicating the relationships between the sets. (www.thefreedictionary.com/**Venn+diagram*)***. Venn-Diagrams are often used to determine the probability of an event.

Let's do a quick example. A group of 30 students were asked to make a Venn-Diagram based on the type of siblings they had (sisters or brothers or both). Five students only had sisters, 16 students only had brothers and 9 students had either a brother or a sister. The first step is to notice that there are only two variables (sister and brother) and therefore there will only be two circles in this Venn-Diagram. The second step is to draw the two overlapping circles and label one sister and one brother. Make sure you put the universe of interest (represented by the rectangle) around the circles. **This has been done for you below.**



The third step is to place how many students have sisters, brothers or both into the circle. Remember that the section of the circle that overlaps means the students have both brothers and sisters. If students were to have no brothers or sisters then you would put them outside of the circles but within the rectangle. **Fill in the number of students with different types of siblings below.**



Congratulations. This is now a Venn-Diagram showing the number of students with different siblings.

We can now find the probability how many students would be likely to have certain sibling types.

Remember the probability equation:

Probability of event = the number in the event / total number of trials

Therefore, the probability of having a sister would be:

P (having a sister) = the number of students with a sister / total number of students

P (having a sister) = 5/30

P (having a sister) = 1/6

Can you determine what would be the probability that a student would have both a sister and a brother?

Now that you have mastered finding probabilities using Venn Diagrams let's find out some probabilities from our 'real world' scenarios.

Venn-Diagram

Q1) Find the probabilities for the chocolate (Mars Bar and Crunchie) Venn-Diagram.

e) P (only Mars Bars)
f) P (Mars Bar and Crunchie)
g) P (only Mars Bar or only Crunchie)

Q2) Find the probabilities for the student mode of transport Venn-Diagram .

a) P (Bus)
b) P (Bus and Car)
c) P (Car or Bike)
d) P (Bus and Car and Bike)
e) P (Car not Bus)
f) P (not Car and not Bus and not Bike)

Excellent. You have found the probability of the above events. All events have a probability range between 0 -1. We can make predictions on the likelihood of an event by looking at where our event falls in the probability range. Below is the probability line, with likelihood of an event below the event value.

0	0.25	0.5	0.75	1
Impossible	Unlikely	Equally likely and unlikely	Likely	Certain

So let us find where our events fall within this probability range. For our example at the start of this worksheet we saw the probability of having a sister was 1/6.

P (having a sister) = 1/6

Another way of representing this would be:

P (having a sister) = 0.16

This is because 1 divided by 6 equals 0.16. On the probability line 0.16 falls between 'Impossible' and 'Unlikely'. We would therefore say that 'The probability of picking a student with a sister is highly unlikely'.

Now it’s your turn. Find where your probability values fall on the probability line for questions 1 and 2. Don’t forget to write your final answer in a sentence – just like I have done in the example above.